# THE POWER OF HOLOMORPHY – EXACT RESULTS IN 4D SUSY FIELD THEORIES

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#### ABSTRACT

Holomorphy of the superpotential and of the coefficient of the gauge kinetic terms in supersymmetric theories lead to powerful results. They are the underlying conceptual reason for the important non-renormalization theorems. They also enable us to study the exact non-perturbative dynamics of these theories. We find explicit realizations of known phenomena as well as new ones in four dimensional strongly coupled field theories. These shed new light on confinement and chiral symmetry breaking. This note is based on a talk delivered at the PASCOS (94) meeting at Syracuse University.

### 1. Introduction

Perhaps the most remarkable property of supersymmetric field theories is the non-renormalization theorem. In this talk we will give a conceptual understanding of the theorem, and extend it beyond perturbation theory. We will identify the holomorphy of the superpotential and of the coefficient of the gauge kinetic term as the crucial elements in this theorem<sup>1</sup>. The non-perturbative understanding will enable us to derive exact results in these strongly coupled four dimensional supersymmetric gauge theories<sup>2,3,4</sup> (see also, a related discussion in reference 5).

There are three main applications to these supersymmetric theories:

- 1. The gauge hierarchy problem: This was the original motivation for studying the dynamics of supersymmetric field theories. First, the perturbative non-renormalization protects a tree level hierarchy. Non-perturbative violation of the perturbative theorem allows dynamical supersymmetry breaking at low energies<sup>6</sup>. The new non-renormalization theorem enables one to analyze the theory exactly.
- 2. Topological Field Theories<sup>7</sup>. We will have little to say about them here except to note that we expect our new dynamical insights to be helpful in understanding them.
- 3. The new application, which we will suggest here, is that they can be used as "toy models" for the dynamics of strongly coupled four dimensional field theories.

Our ability to find exact results allows us to study long standing issues like chiral symmetry breaking and confinement.

For lack of time, we will be rather brief and will mention only the main points here. The talk will be like a quick tour through the different models and phenomena that were found. The interested reader is referred to the original papers for details.

## 2. Non-renormalization Theorem

## 2.1. General Rules

The standard proof<sup>8</sup> of the non-renormalization theorem is based on the study of Feynman diagrams in superspace. Its main drawbacks are its complexity and the limitation to perturbation theory. The new proof which we will present here generalizes observations of references 9, 10 and 11. It is similar in spirit to the non-renormalization in sigma model perturbation theory<sup>12</sup> and in semiclassical perturbation theory<sup>13</sup> in string theory.

We will think of all the coupling constants in the tree level superpotential  $W_{\text{tree}}$ ,  $\lambda_I$ , and the gauge coupling,  $\frac{4\pi}{g^2} \sim \log \Lambda$ , as background fields. Then, the renormalized, effective superpotential,  $W_{\text{eff}}(\phi_i, \lambda_I, \Lambda)$  is constrained by:

- 1. Symmetries: By assigning transformations laws both to the fields and to the coupling constants the theory has a larger symmetry. The effective Lagrangian should be invariant under it. Such restrictions are common in physics and are usually referred to as selection rules.
- 2. Holomorphy:  $W_{\text{eff}}$  is independent of  $\lambda_I^{\dagger}$ . This is the key property. Just as the superpotential is holomorphic in the fields, it is also holomorphic in the coupling constants (the background fields). This is unlike the effective Lagrangian in non-supersymmetric theories which is not subject to any holomorphy restrictions.
- 3. Various limits:  $W_{\text{eff}}$  can be analyzed approximately at weak coupling. The singularities have physical meaning and can be controlled.

Often this enables us to determine  $W_{\text{eff}}$  completely. The point is that a holomorphic function (more precisely, a section) is determined by its asymptotic behavior and singularities.

A crucial subtlety has to be mentioned at this point. As explained by Shifman and Vainshtein<sup>10</sup> there are two different objects which are usually called "the effective action." The 1PI effective action and the Wilsonian one. When there are no interacting massless particles, these two effective actions are identical. However, when interacting massless particles are present, the 1PI effective action suffers from IR ambiguities and might suffer from holomorphic anomalies. These are absent in the Wilsonian effective action.

There are two obvious extensions to our discussion. First, it easily extends to the coefficient  $f(\Phi)$  in  $\int d^2\theta f(\Phi)W_{\alpha}^2$  which is also holomorphic. Second, in N=2

supersymmetry the Kahler potential is related by supersymmetry to such an  $f(\Phi)$  and therefore it is also constrained by our considerations.

## 2.2. Example: Wess-Zumino Model

In order to demonstrate our rules we study here the simplest Wess-Zumino model and rederive the known non-renormalization theorem. Consider the theory based on the tree level superpotential

$$W_{\text{tree}} = m\phi^2 + \lambda\phi^3$$
.

We will make use of two U(1) symmetries. The charges of the field  $\phi$  and the coupling constants m and  $\lambda$  are

$$\begin{array}{cccc} & U(1) & \times & U(1)_R \\ \phi & 1 & & 1 \\ m & -2 & & 0 \\ \lambda & -3 & & -1 \end{array}$$

where  $U(1)_R$  is an R symmetry. Note that non-zero values for  $\lambda$  and m explicitly break both of them. However, the symmetry still leads to selection rules.

The symmetries and holomorphy of the effective superpotential restrict it to be of the form

$$W_{\text{eff}} = m\phi^2 f\left(\frac{\lambda\phi}{m}\right).$$

We proceed, according to our general rules above, by studying the limit of small  $\lambda$ . In this region perturbation theory is valid and the superpotential can be expanded as

$$W_{\text{eff}} = \sum_{n=0}^{\infty} a_n \frac{1}{m^{n-1}} \lambda^n \phi^{n+2}.$$

The *n*'th term in this expansion can arise from a tree diagram with n+2 external legs, n vertices and n-1 propagators. For  $n \geq 2$  it is not 1PI and its contribution should not be included in the effective action. Higher order corrections in  $\lambda$  with the same number of external legs could arise from loop diagrams. However, they must be absent because they are not compatible with the form of  $W_{\text{eff}}$ .

We conclude that the effective superpotential is

$$W_{\text{eff}} = m\phi^2 + \lambda\phi^3 = W_{\text{tree}};$$

i.e. the superpotential is not renormalized.

Thus, we rederive the standard perturbative non-renormalization theorem. Furthermore, our result extends it beyond perturbation theory. Strictly speaking, the Wess-Zumino model probably does not exist as an interacting quantum field theory in four dimensions and therefore this non-perturbative result is of little interest. However, such a model does exist in two dimensions and there our non-perturbative result applies.

If there are several fields, some of them are heavy and the other are light, the heavy fields can be integrated out to yield a low energy effective Lagrangian for the light fields. Then, the contribution of tree diagrams with intermediate heavy fields should be included in the effective action. As we saw, our simple rules allow such diagrams to contribute. Our result is thus compatible with the known tree level renormalization of the superpotential.

# 3. Applications To N = 1 SUSY Gauge Theories

# 3.1 Supersymmetric QCD - Classical Moduli Space

We now turn to more complicated models which are asymptotically free and probably exist non-perturbatively. Consider a supersymmetric  $SU(N_c)$  gauge theory with  $N_f$  quark flavors in the fundamental representation. The elementary fields are gluons, quarks and antiquarks:

$$W_{\alpha} = \lambda_{\alpha} + \theta_{\beta} \sigma_{\alpha}^{\mu\nu\beta} F_{\mu\nu} + \dots$$

$$Q_{i} = q_{i} + \theta \psi_{i} + \dots \qquad \qquad i, \tilde{i} = 1, \dots, N_{f}$$

$$\tilde{Q}^{\tilde{i}} = \tilde{q}^{\tilde{i}} + \theta \tilde{\psi}^{\tilde{i}} + \dots$$

An important property of these theories is that in the absence of mass terms they have a continuum of classical ground states – flat directions. These are directions in field space along which the classical potential vanishes. Unlike the situation with spontaneous symmetry breaking, these are physically inequivalent ground states. In string theory such a continuum of inequivalent ground states is generic. Borrowing the terminology from string theory, we will refer to the space of classical ground states as the "classical moduli space." More explicitly, up to gauge and global symmetry transformations they are given by:

$$q = \tilde{q} = \begin{pmatrix} a_1 & & & \\ & a_2 & & \\ & & \cdot & \\ & & a_{N_f} \end{pmatrix}$$

for  $N_f < N_c$  and by

$$q = \begin{pmatrix} a_1 & & & \\ & a_2 & & \\ & & \ddots & \\ & & & a_{N_c} \end{pmatrix}; \quad \tilde{q} = \begin{pmatrix} \tilde{a}_1 & & & \\ & \tilde{a}_2 & & \\ & & \ddots & \\ & & & \tilde{a}_{N_c} \end{pmatrix}$$

$$|a_i|^2 - |\tilde{a}_i|^2 = \text{independent of } i$$

for  $N_f \geq N_c$ .

Already at the classical level we can integrate out the massive fields and consider an effective Lagrangian for the massless modes. Their expectation values label the particular ground state we expand around, and hence they are coordinates on the classical moduli space. We will call them moduli. The classical moduli space is not smooth. Its singularities are at the points of enhanced gauge symmetry. For instance, when  $a_i = \tilde{a}_i = 0$  for every i the gauge symmetry is totally unbroken. Therefore, the low energy effective theory of the moduli is singular there. This should not surprise us. At these singular points there are new massless particles—gluons. An effective Lagrangian without them is singular. If we include them in the low energy description, the Lagrangian is smooth.

In the next subsections we will see how this picture changes in the quantum theory. At large field strength, far from the classical singularities the gauge symmetry is broken at a high scale. Therefore, the quantum theory is weakly coupled and semiclassical techniques are reliable. We expect the quantum corrections to the classical picture to be small there. On the other hand, at small field strength the quantum theory is strongly coupled and the quantum corrections can be large and cause dramatic modifications to the classical behavior.

3.2. 
$$N_f < N_c - No \ Vacuum$$

The first question to ask is whether the classical vacuum degeneracy can be removed quantum mechanically. For that to happen a superpotential will have to be generated. There is a unique invariant superpotential<sup>14</sup>

$$W_{\text{eff}} = (N_c - N_f) \frac{\Lambda^{\frac{3N_c - N_f}{N_c - N_f}}}{\left(\det \tilde{Q}Q\right)^{\frac{1}{N_c - N_f}}}$$

where  $\Lambda$  is the dynamically generated scale of the theory. The prefactor  $N_c - N_f$  is a choice of normalization of  $\Lambda$  (a choice of subtraction scheme). Therefore, if the vacuum degeneracy is removed, this particular superpotential must be generated. For  $N_f \geq N_c$  this superpotential does not exist (either the exponent diverges or the determinant vanishes) and therefore the vacuum degeneracy cannot be lifted. We will return to this case in the next subsections.

For  $N_f < N_c$  this superpotential is generated dynamically<sup>6</sup>. For  $N_f = N_c - 1$  it is generated by instantons and for  $N_f < N_c - 1$  by gluino condensation. (For related work on this model see also references 9 and 15.) This dynamically generated superpotential leads to a potential which slopes to zero at infinity. Therefore, the quantum theory does not have a ground state. We started with an infinite set of vacua in the classical theory and ended up in the quantum theory without a vacuum!

3.3. 
$$N_f = N_c - Quantum \ Moduli \ Space$$

As we said above, for  $N_f \geq N_c$  the vacuum degeneracy is not lifted. Therefore, the quantum theory also has a continuous space of inequivalent vacua. Since this space can be different than the classical one, we will refer to it as the "quantum moduli space." The most interesting questions about it are associated with the nature of its singularities. Classically, the singularities were associated with massless gluons. Are

there singularities in the quantum moduli space? what are the massless particles in those singularities?

In order to answer these questions we start with the first case of  $N_f = N_c$  and give a gauge invariant description of the moduli space. Consider the mesons, baryon and anti-baryon composite fields

$$M_i^{\tilde{i}} = \tilde{Q}^{\tilde{i}} Q_i$$

$$B = Q^{N_c}$$

$$\tilde{B} = \tilde{Q}^{N_c}.$$

Classically

$$\det M - \tilde{B}B = 0$$

which follows from Bose statistics of Q and  $\tilde{Q}$ . It is easy to see that the classical moduli space is the space of M, B and  $\tilde{B}$  subject to this relation.

We will not give the derivation here but will simply assert that the quantum moduli space is parameterized by the same fields but the constraint is modified<sup>2</sup> to

$$\det M - \tilde{B}B = \Lambda^{2N_c} \neq 0.$$

It is important that this space is smooth. There are no singularities in the quantum moduli space. All the singularities have been smoothed out. (For a simple low dimensional example of how such a singularity can be smoothed out consider an hyperboloid in a cone. Far from the tip of the cone the two spaces are similar, but the singularity at the tip of the cone is smoothed out on the hyperboloid.)

Since there are no singularities on the quantum moduli space, the only massless particles are the moduli. The gluons are "Higgsed" in the semiclassical region of large fields and are confined for small fields. Note that there is a smooth transition from a region where a Higgs description is more appropriate to a region where a confining description is more appropriate. This is possible because of the presence of matter fields in the fundamental representation of the gauge group<sup>16</sup>.

Different points on the quantum moduli space exhibit different patterns of global symmetry breaking. For example at  $M_i^{\tilde{i}} = \Lambda^2 \delta_i^{\tilde{i}}$ ,  $B = \tilde{B} = 0$  the symmetry is broken as

$$SU(N_f)_L \times SU(N_f)_R \times U(1)_B \times U(1)_R \to SU(N_f)_V \times U(1)_B \times U(1)_R.$$

At M=0,  $B=-\tilde{B}=\Lambda^{N_c}$  the breaking pattern is

$$SU(N_f)_L \times SU(N_f)_R \times U(1)_B \times U(1)_R \rightarrow SU(N_f)_L \times SU(N_f)_R \times U(1)_R$$
.

In both cases some of the moduli are Goldstone bosons of the broken symmetry. It is straightforward to check that the massless fermions saturate the 'tHooft anomaly conditions for the unbroken symmetries.

3.4.  $N_f = N_c + 1$  – Confinement Without Chiral Symmetry Breaking

We now add another flavor to the previous case. The classical moduli space is again described by the mesons M, baryons B and anti-baryons  $\tilde{B}$  subject to the

constraints

$$\begin{split} \det M \left(\frac{1}{M}\right)_{\tilde{i}}^{i} - \tilde{B}_{\tilde{i}}B^{i} &= 0 \\ M_{i}^{\tilde{i}}B^{i} &= \tilde{B}_{\tilde{i}}M_{i}^{\tilde{i}}. \end{split}$$

Unlike the previous case, here the quantum moduli space can be shown to be the same as the classical one<sup>2</sup>. Therefore, it is singular and we should interpret the singularities in the quantum theory.

It turns out<sup>2</sup> that in the quantum theory all the degrees of freedom in M, B and  $\tilde{B}$  are physical and they couple through the superpotential

$$W_{\text{eff}} = \frac{1}{\Lambda^{2N_c - 1}} (\tilde{B}_{\tilde{i}} M_i^{\tilde{i}} B^i - \det M).$$

The classical constraints appears as the equations of motion  $\frac{\partial W_{\text{eff}}}{\partial M} = \frac{\partial W_{\text{eff}}}{\partial B} = \frac{\partial W_{\text{eff}}}{\partial B} = 0$ . The singularities, however, are interpreted differently than in the classical theory. At the point  $M = \tilde{B} = B = 0$  the global chiral symmetry

$$SU(N_f)_L \times SU(N_f)_R \times U(1)_B \times U(1)_R$$

is unbroken and all the components of M, B and  $\tilde{B}$  are massless. Again, it is easy to check that the 'tHooft anomaly conditions are saturated.

We conclude that the spectrum at the origin of field space consists of massless composite mesons and baryons and the chiral symmetry of the theory is unbroken there. This is confinement without chiral symmetry breaking. Again, we see a smooth transition<sup>16</sup> from the semiclassical region where a Higgs description is more appropriate to a strongly coupled region where a confining description is more appropriate.

3.4. 
$$N_f > N_c + 1$$
 – No Confinement; Interacting Superconformal Theories

In this subsection we continue to add flavors and discuss the situation of  $N_f > N_c + 1$ . As for  $N_f = N_c + 1$ , the classical moduli space is unchanged quantum mechanically<sup>2</sup>. Therefore, there are singularities in the quantum moduli space and new massless states should appear there. However, it can be shown that it is impossible to interpret the fields which create them as gauge invariant polynomials in the elementary fields. Therefore, we conjecture that at the singularities the fundamental quarks and the gluons are massless.

According to standard quantum field theory, the theory of the massless states must be scale invariant. In the previous cases this theory was free. Now, on the other hand, the massless fields interact, so the low energy theory must be a non-trivial scale invariant theory in which the beta function vanishes.

Such interacting scale invariant theories in four dimensions are known to exist for large  $N_c$  and  $N_f$ , by balancing the first two terms in the beta function. The resulting fixed point is at weak coupling thus justifying the expansion. More explicitly, we can show that this fixed point exists for  $N_f, N_c \to \infty$  with  $\epsilon = \frac{3N_c - N_f}{N_c}$  small. We conjecture that such a fixed point persists for small  $N_f$  and  $N_c$  for every  $N_f > N_c + 1$ .

We conclude that at the singular points on the moduli space the theory is at non-trivial fixed points of the renormalization group; i.e. these are interacting superconformal field theories.

In such a scale invariant theory the beta function vanishes but the operators can have non-canonical dimensions. We cannot compute all these dimensions. However, for chiral superfields the dimensions are determined as follows. The superconformal algebra in four dimensions includes a  $U(1)_R$  symmetry. The dimensions of chiral fields, D, are related to their R charge by

$$D = \frac{3}{2}R.$$

Around the UV (semiclassical) fixed point there is only one anomaly free R symmetry which commutes with all the flavor symmetries. Therefore, this must be the symmetry in the superconformal algebra at the non-trivial fixed point. We conclude that 2,17

$$D(\tilde{Q}Q) = 3\frac{N_f - N_c}{N_f}.$$

## 3.5. Non-trivial Superpotentials

In the previous examples the superpotential and/or the structure of the moduli space was essentially determined by the symmetries, holomorphy and the weak coupling asymptotics. Correspondingly, in all these models  $W_{\rm eff}$  is a single power of the fields which is fixed by the symmetries. More complicated models, where more powerful techniques are needed, were studied in reference 3.

Consider, for a example, a model based on the gauge group  $SU(2)_1 \times SU(2)_2$  with matter fields transforming like

$$egin{array}{ll} Q & ({f 2},{f 2}) \\ L_+ & ({f 1},{f 2}) \\ L_- & ({f 1},{f 2}). \end{array}$$

The classical moduli space is two complex dimensional (at the generic point in the moduli space the gauge group is completely broken, hence six out of the eight elementary chiral matter fields acquire a mass and the other two remain massless). It can be parametrized by the gauge invariant order parameters

$$X = Q^2$$
$$Y = L_+ L_-.$$

The classical singularities are at the points where either X = 0 where  $SU(2)_1$  is unbroken, or Y = 0 where a diagonal subgroup of  $SU(2)_1 \times SU(2)_2$  is unbroken.

Symmetries and holomorphy constrain<sup>3</sup> the effective superpotential to be of the form

$$W_{\text{eff}} = \frac{\Lambda_1^5}{X} f\left(\frac{\Lambda_2^4}{XY}\right)$$

( $\Lambda_1$  and  $\Lambda_2$  are the dynamically generated scales of the two gauge groups) but do not determine the function f. At weak coupling (large XY) the gauge symmetry is completely broken and the superpotential can be generated by instantons. Hence

$$W_{\text{eff}} = \frac{\Lambda_1^5}{X} \sum_{n=0}^{\infty} a_n \left(\frac{\Lambda_2^4}{XY}\right)^n$$

where the n'th term in the sum can arise from instantons with instanton numbers (1,n) under the two gauge groups.

A more detailed analysis is needed in order to determine the coefficients  $a_n$ . The main point<sup>3</sup> is that the classical singularity at XY = 0 moves in the quantum theory to  $XY = \Lambda_2^4$ . Examining the behavior of  $W_{\text{eff}}$  near the point where the argument of  $f\left(\frac{\Lambda_2^4}{XY}\right)$  equals one leads to

$$W_{\rm eff} = \frac{\Lambda_1^5 Y}{XY - \Lambda_2^4} = \sum_{n=0}^{\infty} \frac{\Lambda_1^5}{X} \left(\frac{\Lambda_2^4}{XY}\right)^n.$$

We see that the combination of symmetries, holomorphy, and asymptotic behavior at weak coupling and near the singularities enables us to sum up the instanton series and to find a non-trivial answer.

## 4. Applications To N = 2 SUSY Gauge Theory

In the previous sections we demonstrated the power of holomorphy in determining the superpotential and the topology of the quantum moduli space. However, we were unable to determine the metric on the moduli space. The reason for that is that the metric on the space  $(ds)^2 = g_{i\bar{j}} d\phi^i d\bar{\phi}^{\bar{j}}$  is given by the kinetic term of the moduli

$$g_{i\bar{j}}(\phi,\bar{\phi})\partial_{\mu}\phi^{i}\partial^{\mu}\bar{\phi}^{\bar{j}}$$

which is derived from the Kahler potential

$$g_{i\bar{j}}(\phi,\bar{\phi}) = \partial_i \bar{\partial}_{\bar{j}} K(\phi,\bar{\phi}).$$

Since K is not holomorphic, it is not constrained by the previous considerations. However, in N=2 theories the Kahler potential is controlled by a holomorphic function and therefore it can be determined<sup>4</sup>.

To be more specific, consider an SU(2) gauge theory coupled to a single chiral matter field,  $\Phi$ , in the adjoint representation. This theory has N=2 supersymmetry.

The classical moduli space is one complex dimensional corresponding to the flat direction of the potential

$$\langle \Phi \rangle = \frac{1}{2} \begin{pmatrix} a & 0 \\ 0 & -a \end{pmatrix}.$$

It can be labeled by the gauge invariant order parameter  $u = \langle \text{Tr } \Phi^2 \rangle = \frac{1}{2}a^2$ . This expectation value breaks the SU(2) gauge symmetry to U(1). Therefore, the light

fields along this direction are a photon and its N=2 superpartners – a Dirac fermion and a complex scalar. The expectation value of this scalar a is a local coordinate on the classical moduli space.  $u=\frac{1}{2}a^2$  is a good global coordinate.

The low energy effective action is determined by a single holomorphic function, the prepotential,  $\mathcal{F}(A)$ ,

$$\frac{1}{4\pi} \operatorname{Im} \left[ \int d^4 \theta \frac{\partial \mathcal{F}(A)}{\partial A} \bar{A} + \int d^2 \theta \frac{1}{2} \frac{\partial^2 \mathcal{F}(A)}{\partial A^2} W_{\alpha} W^{\alpha} \right]$$

where A is an N=1 chiral multiplet and  $W_{\alpha}$  is the N=1 field strength of a photon. They combine into a single N=2 vector multiplet  $\mathcal{A}$  whose scalar component is a. This is the flat space limit of "special geometry." <sup>18</sup>

The leading quantum corrections to the metric on the moduli space were studied in reference 19

$$\mathcal{F} = i \frac{1}{2\pi} \mathcal{A}^2 \ln \frac{\mathcal{A}^2}{\Lambda^2} + \sum_{k=1}^{\infty} \mathcal{F}_k \left(\frac{\Lambda}{\mathcal{A}}\right)^{4k} \mathcal{A}^2$$

where the k'th term in the sum represents the contribution of k instantons.

In reference 4 the exact theory was analyzed. A crucial point is the fact that the weak coupling description above is not valid globally. The coordinate a and the function  $\mathcal{F}$  are not single valued on the moduli space. Instead, the pair

$$\begin{pmatrix} A_D = \frac{\partial \mathcal{F}(A)}{\partial A} \\ A \end{pmatrix}$$

is a section of an  $SL(2, \mathbb{Z})$  bundle over the moduli space. More physically, there is freedom to perform duality transformations on the light fields. They are generated by

$$T = \begin{pmatrix} 1 & 1 \\ 0 & 1 \end{pmatrix}$$
$$S = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}.$$

The N=1 gauge multiplet  $W_{\alpha}$  is invariant under T and undergoes a standard electric-magnetic duality transformation under S. The pair  $(A_D = \frac{\partial \mathcal{F}(A)}{\partial A}, A)$  transforms as a column vector which is multiplied by the matrices T or S. These two generators generate the group SL(2,Z). As we move around a closed path on the moduli space, the light fields are not single valued; they can be transformed by an SL(2,Z) transformation.

At large field strength the semiclassical expression above shows that  $a_D \approx i \frac{1}{\pi} a \ln a^2$  where  $a_D$  is the scalar component of the superfield  $A_D$ . In terms of the gauge invariant coordinate  $u \approx \frac{1}{2}a^2$  our pair is

$$\begin{pmatrix} a_D \approx i \frac{1}{\pi} \sqrt{2u} \ln u \\ a \approx \sqrt{2u} \end{pmatrix}.$$

As we move around the moduli space  $u \to e^{2\pi i}u$  the pair transforms by

$$\mathcal{M}_{\infty} = PT^{-2} = \begin{pmatrix} -1 & 2\\ 0 & -1 \end{pmatrix}$$

(P = -1) is the charge conjugation matrix). We see that the pair is not single valued even in the weak coupling region.

It turns out<sup>4</sup> that the quantum moduli space is the complex u plane with two singular points at  $u = \langle \text{Tr } \Phi^2 \rangle = \pm 1$  (after rescaling the value of u by a factor proportional to  $\Lambda^2$ ). The pair  $(a_D, a)$  transforms by

$$\mathcal{M}_1 = ST^2S^{-1} = \begin{pmatrix} 1 & 0 \\ -2 & 1 \end{pmatrix}$$

as we circle around the singularity at u = 1 and by

$$\mathcal{M}_{-1} = (TS)T^2(TS)^{-1} = \begin{pmatrix} -1 & 2 \\ -2 & 3 \end{pmatrix}$$

as we circle around u = -1. Using this information the pair  $(a_D(u), a(u))$  can be found<sup>4</sup>

$$a_D(u) = \frac{\sqrt{2}}{\pi} \int_1^u \frac{dx \sqrt{x - u}}{\sqrt{x^2 - 1}}$$
$$a(u) = \frac{\sqrt{2}}{\pi} \int_{-1}^1 \frac{dx \sqrt{x - u}}{\sqrt{x^2 - 1}}.$$

The Kahler potential which is  $K = \text{Im } a_D(u)\bar{a}(\bar{u})$  is therefore also known and so is the metric derived from it.

As in our previous examples the singularities on the moduli space are associated with new massless particles. In this case these are massless magnetic monopoles<sup>4</sup>. The low energy effective theory around these points is non-local in terms of the elementary photon. However, the dual of the photon, "the magnetic photon," couples locally to the massless magnetic monopoles. Therefore, the low energy theory looks like N=2 supersymmetric QED with massless charged fields (the monopoles).

The knowledge of  $(a_D(u), a(u))$  enables us to find the masses of the stable BPS-saturated<sup>20</sup> states. As explained by Witten and Olive<sup>21</sup>, their masses are related to a central extension in the N=2 algebra. Including the quantum corrections the masses are expressed in terms of the magnetic and electric charges of the particles  $(n_m, n_e)$  as<sup>4</sup>

$$M = \sqrt{2}|Z|$$
 with  $Z = n_m a_D(u) + n_e a(u)$ .

We can break N=2 supersymmetry to N=1 by adding a mass term to the chiral superfield  $\Phi$ ,  $W=m{\rm Tr}\;\Phi^2$ . Then, the following things happen<sup>4</sup>:

1. The classical moduli space becomes one point  $\Phi = 0$  where all the elementary fields are massless. The quantum moduli space collapses to the two singular points with  $u = \langle \text{Tr } \Phi^2 \rangle = \pm 1$ . Since the value of u is non-zero at these points, the chiral  $Z_2$  symmetry of the theory is spontaneously broken.

2. The massless monopoles at these points condense. This condensation gives a mass to the previously massless photon by the Higgs mechanism and a mass gap is generated. However, since this photon is dual to the electric photon, what we see here is actually confinement of electric charges.

For  $m \gg \Lambda$ , the elementary chiral field  $\Phi$  can be integrated out at high energies. The low energy theory is the N=1 supersymmetric Yang-Mills theory without matter fields. This theory is expected to confine and to have two ground states with a mass gap in which its discrete  $Z_2$  chiral symmetry is spontaneously broken<sup>22</sup>. These are precisely the phenomena we saw above.

## 5. Conclusions

We showed that holomorphy is the principle underlying the non-renormalization theorem. It enables us to control supersymmetric strongly coupled field theories and to find exact results.

These strongly coupled theories exhibit a large number of interesting phenomena:

- 1. Quantum moduli spaces
- 2. Smoothed out singularities
- 3. Non-conventional patterns of chiral symmetry breaking (not most attractive channel)
- 4. Massless composite mesons and baryons
- 5. Lack of confinement and interacting CFT in four dimensions
- 6. Massless magnetic monopoles
- 7. Monopole condensation as mechanism for confinement
- 8. Calculable non-trivial terms in the effective Lagrangian
- 9. Calculable spectrum of stable particles

We expect that further explorations of these theories will teach us even more about the three applications we mentioned in the introduction (dynamical supersymmetry breaking, topological field theories and the dynamics of more generic strongly coupled four dimensional field theories).

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